Simplify each expression.

1. $\dfrac{\sec(x)\sin(x)}{\tan(x)+\cot(x)}$

$$= \frac{\frac{1}{\cos x}\cdot\frac{\sin x}{1}\cdot\cos x}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\cdot\cos x} = \frac{\sin x\cdot\sin x}{\sin x+\frac{\cos^2 x}{\sin x}\cdot\sin x}$$

$$= \frac{\sin^2 x}{\sin^2 x+\cos^2 x} = \frac{\sin^2 x}{1} = \boxed{\sin^2 x}$$

2. $\dfrac{\sec(x)}{\cos(x)}-\dfrac{\tan(x)}{\cot(x)}$

$$= \frac{\left(\frac{1}{\cos x}\right)}{\cos x}-\frac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{\cos x}{\sin x}\right)} = \frac{1}{\cos^2 x}-\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\cos x}$$

$$= \frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x} = \frac{1-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = \boxed{1}$$

3. $\csc^2 x\tan^2 x-1$

$$= \frac{1}{\sin^2 x}\cdot\frac{\sin^2 x}{\cos^2 x}-1 = \frac{1}{\cos^2 x}-1 = \sec^2 x-1$$

$$= \boxed{\tan^2 x}$$

$\dfrac{\sin^2 x}{\cos^2 x}+\dfrac{\cos^2 x}{\cos^2 x}=\dfrac{1}{\cos^2 x}$

$\tan^2 x+1=\sec^2 x$

4. $(\sin x+\cos x)^2+(\sin x-\cos x)^2$

$$= \sin^2 x+2\sin x\cos x+\cos^2 x+\sin^2 x-2\sin x\cos x+\cos^2 x$$

$$= 2\sin^2 x+2\cos^2 x$$

$$= 2(\sin^2 x+\cos^2 x)$$

$$= 2(1) = \boxed{2}$$

5. $(\sin x+\cos x)(\tan x+\cot x)$

$$= \sin x\tan x+\sin x\cot x+\cos x\tan x+\cos x\cot x$$

$$= \sin x\cdot\frac{\sin x}{\cos x}+\sin x\cdot\frac{\cos x}{\sin x}+\cos x\cdot\frac{\sin x}{\cos x}+\cos x\cdot\frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x}+\frac{\cos x}{1}\cdot\frac{\cos x}{\cos x}+\frac{\sin x}{1}\cdot\frac{\sin x}{\sin x}+\frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x}+\frac{\cos^2 x}{\cos x}+\frac{\sin^2 x}{\sin x}+\frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x+\cos^2 x}{\cos x}+\frac{\sin^2 x+\cos^2 x}{\sin x}$$

$$= \frac{1}{\cos x}+\frac{1}{\sin x} = \boxed{\sec x+\csc x}$$

6. $\dfrac{\sin(x)+\tan(x)}{1+\sec(x)}$

$$= \frac{\frac{\sin x}{1}+\frac{\sin x}{\cos x}\cdot\cos x}{1+\frac{1}{\cos x}\cdot\cos x}$$

$$= \frac{\sin x\cos x+\sin x}{\cos x+1} = \frac{\sin x(\cos x+1)}{(\cos x+1)} = \boxed{\sin x}$$