

Simplify each expression.

$$\begin{aligned}
 1. \frac{\sec(x) \sin(x)}{\tan(x) + \cot(x)} &= \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{1} \cdot \cos x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \cos x} = \frac{\sin x}{\sin x + \frac{\cos^2 x}{\sin x}} \cdot \sin x \\
 &= \frac{\sin^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x}{1} = \boxed{\sin^2 x}
 \end{aligned}$$

$$\begin{aligned}
 2. \frac{\sec(x)}{\cos(x)} - \frac{\tan(x)}{\cot(x)} &= \frac{\left(\frac{1}{\cos x}\right)}{\cos x} - \frac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{\cos x}{\sin x}\right)} = \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\cos x} \\
 &= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = \boxed{1}
 \end{aligned}$$

$$\begin{aligned}
 3. \csc^2 x \tan^2 x - 1 &= \frac{1}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} - 1 = \frac{1}{\cos^2 x} - 1 = \sec^2 x - 1 \\
 &= \boxed{\tan^2 x}
 \end{aligned}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$+ \tan^2 x + 1 = \sec^2 x$$

$$\begin{aligned}
 4. (\sin x + \cos x)^2 + (\sin x - \cos x)^2 &= \sin^2 x + 2 \cancel{\sin x \cos x + \cos^2 x} + \\
 &\quad \cancel{\sin^2 x - 2 \sin x \cos x + \cos^2 x} \\
 &= 2 \sin^2 x + 2 \cos^2 x \\
 &= 2(\sin^2 x + \cos^2 x) \\
 &= 2(1) = \boxed{2}
 \end{aligned}$$

$$5. (\sin x + \cos x)(\tan x + \cot x)$$

$$= \sin x + \tan x + \sin x \cot x + \cos x + \tan x + \cos x \cot x$$

$$= \sin x \cdot \frac{\sin x}{\cos x} + \cancel{\sin x} \cdot \frac{\cos x}{\sin x} + \cancel{\cos x} \cdot \frac{\sin x}{\cos x} + \cos x \cdot \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos x \cdot \cos x}{1} \cdot \frac{\cos x}{\cos x} + \frac{\sin x}{1} \cdot \frac{\sin x}{\sin x} + \frac{\cos^2 x}{\sin x}$$

$$6. \frac{\sin(x) + \tan(x)}{1 + \sec(x)}$$

$$= \frac{\sin x}{1} + \frac{\sin x}{\cos x} \cdot \cos x$$

$$= 1 + \frac{1}{\cos x} \cdot \cos x$$

$$= \frac{\sin x \cos x + \sin x}{\cos x + 1}$$

$$\begin{aligned}
 &= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} \\
 &= \frac{\sin^2 x + \cos^2 x}{\cos x} + \frac{\sin^2 x + \cos^2 x}{\sin x} \\
 &= \frac{1}{\cos x} + \frac{1}{\sin x} = \boxed{\sec x + \csc x}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sin x (\cos x + 1)}{(\cos x + 1)} = \boxed{\sin x}
 \end{aligned}$$