Verify each equation.

1. $\sec x - \tan x \sin x = \dfrac{1}{\sec x}$

$$= \frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1}$$ • recip/quotient Ids

$$= \frac{1 - \sin^2 x}{\cos x}$$ • common denom

$$= \frac{\cos^2 x}{\cos x}$$ • Pyth

$$= \cos x = \frac{1}{\sec x} \checkmark$$ • Cancelling + recip. Id.

2. $\dfrac{1 + \cos x}{\sin x} = \csc x + \cot x$

$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$ • Common denom

$$= \csc x + \cot x \checkmark$$ • Recip. Id.

3. $\cos^2 x - \sin^2 \cancel{x} = 1 - 2\sin^2 x$

$$= 1 - \sin^2 x - \sin^2 x$$ • Rewrite

$$= \cos^2 x - \sin^2 x \checkmark$$ • Pyth

4. $\dfrac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$
$$\tan^2 x + 1 = \sec^2 x$$

$$= \frac{\sec^2 x}{\tan^2 x}$$ • Pyth

$$= \frac{\left(\frac{1}{\cos^2 x}\right) \cdot \cancel{\cos^2 x}}{\left(\frac{\sin^2 x}{\cos^2 x}\right) \cdot \cancel{\cos^2 x}} = \frac{1}{\sin^2 x} = \csc^2 x \checkmark$$ • Recip Id

5. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$$= \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x}{1} \cdot \frac{\cos^2 x}{\cos^2 x}$$ • Quotient Id • $\cos^2 x$

$$= \frac{\sin^2 x - \sin^2 x \cdot \cos^2 x}{\cos^2 x}$$ • Common denom

$$= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$$ • Factor

$$= \frac{\sin^2 x \cdot \sin^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = \tan^2 x \cdot \sin^2 x \checkmark$$
• Pyth • Unmultiply • Quotient Id

6. $\dfrac{\tan x - 1}{\tan x + 1} = \dfrac{1 - \cot x}{1 + \cot x}$

$= \dfrac{\tan x \cdot \cot x - \cot x}{\tan x \cdot \cot x + \cot x}$

$= \dfrac{\dfrac{\tan x}{1} \cdot \dfrac{1}{\tan x} - \cot x}{\dfrac{\tan x}{1} \cdot \dfrac{1}{\tan x} + \cot x}$ • Recip Id

$= \dfrac{1 - \cot x}{1 + \cot x}$ ✓

7. $\dfrac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$

$= \dfrac{1 - \tan^2 x}{\sec^2 x}$ • Pyth Id

$= \left(1 - \dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x$ • Recip + quotient Id

$= \cos^2 x - \sin^2 x$ • Distribute

$= 1 - \sin^2 x - \sin^2 x$ • Pyth Id

$= 1 - 2\sin^2 x$ ✓   • CLT

8. $\dfrac{\cos x + 1}{\sin^3 x} = \dfrac{\csc x}{1 - \cos x}$

$= \dfrac{\cos x + 1}{\sin^2 x \cdot \sin x}$ • Factor

$= \dfrac{\cos x + 1}{(1 - \cos^2 x)\sin x}$ • Pyth Id.

$= \dfrac{(\cos x + 1)}{(1 + \cos x)(1 - \cos x) \cdot \sin x}$ • Factor + cancel

$= \dfrac{1}{(1 - \cos x)} \cdot \dfrac{1}{\sin x}$ • Unmultiply

$= \dfrac{\csc x}{1 - \cos x}$ ✓   • Quotient Id.

9. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

$= (\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x)$ • Diff. of squares

$= (\csc^2 x + \cot^2 x) \cdot 1$ • Pyth. Id

$= \csc^2 x + \cot^2 x$ ✓   • Simplify

$\dfrac{\sin^2 x + \cos^2 x = 1}{\sin^2 x} \quad \dfrac{}{\sin^2 x} \quad \dfrac{}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$1 = \csc^2 x - \cot^2 x$

10. $\dfrac{\tan x}{\sec x} + \dfrac{\cot x}{\csc x} = \sin x + \cos x$

$= \dfrac{\left(\dfrac{\sin x}{\cos x}\right) \cdot \cos x}{\left(\dfrac{1}{\cos x}\right) \cdot \cos x} + \dfrac{\left(\dfrac{\cos x}{\sin x}\right) \cdot \sin x}{\left(\dfrac{1}{\sin x}\right) \cdot \sin x}$ • Recip. + Quotient Ids

$= \dfrac{\sin x}{1} + \dfrac{\cos x}{1}$ • Clear denoms

$= \sin x + \cos x$ ✓   • Simplify