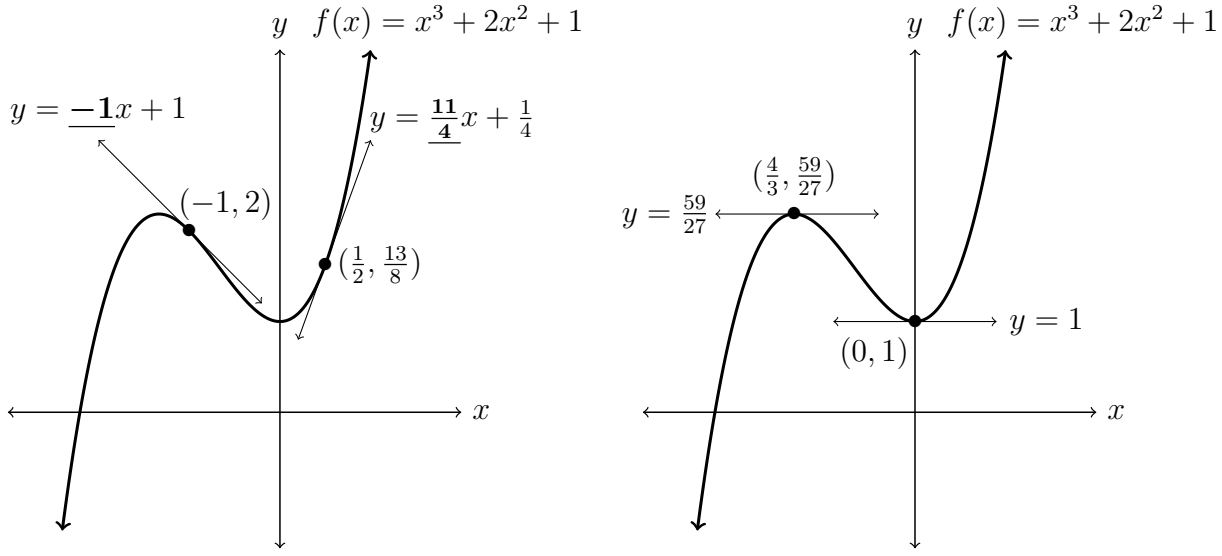
An Introduction to Derivatives

A derivative of a function $f(x)$ can be thought of as a separate, special function which describes change in or "slope" of the original function. The derivative of $f(x)$ is referred to as $\boldsymbol{f'(x)}$ (pronounced "f prime of x").

For any point of $f(x)$, there is a "tangent line" which barely touches $f(x)$ at that point. The slopes of these tangent lines at each point give the value of the derivative function $\boldsymbol{f'(x)}$ at that point.



Notice: When the graph is "smooth" and has a maximum or a minimum point, the tangent line is horizontal, so the slope is 0. **When $\boldsymbol{f'(x) = 0}$, the corresponding point in $\boldsymbol{f(x)}$ is a potential maximum or minimum point.**

Trig Function Project

Assignment: You may work with a partner. Answers must be in complete sentences when appropriate.

You will be working with three functions.

$$g(x) = 3\sin\left(2x + \frac{\pi}{2}\right)$$

$$h(x) = 2\cos\left(4x + \pi\right)$$

$$k(x) = 3\sin\left(2x + \frac{\pi}{2}\right) + 2\cos\left(4x + \pi\right) + 10$$

Notice: $k(x) = g(x) + h(x) + 10$

Grading: Each group may submit a **single** assignment. This project is worth a total of **40 points**.

- **Part 1** - 20 points.
- **Part 2** - 10 points.
- **Part 3** - 10 points.

Due date: Your completed assignment is due on **Tuesday, March 17th**.

Part 1 — For both $g(x)$ and $h(x)$:

1. Find the domain and range.
2. Find the amplitude and period.
3. Find the phase shift.
4. Draw an accurate graph of the function *by hand* on graph paper. Include at least two periods.
5. Find the maximum and minimum points for **one period**. Then give general solutions for when these occur.

Part 2 — For $k(x)$:

1. Use **desmos.com** or another graphing utility to print an accurate graph of the function. Include at least two periods.

2. Use your graph to find the period of $k(x)$. Explain your reasoning. How does the period of $k(x)$ compare to the periods of $g(x)$ and $h(x)$?

3. Use your graph to *estimate* the maximum and minimum points of $k(x)$ for **one period**.

The *derivative* of $k(x)$ is
$\boldsymbol{k'(x)} = 6\cos\left(2x + \frac{\pi}{2}\right) - 8\sin(4x + \pi)$
You will learn how to compute this derivative yourself in Math 1A or Math 12.
For now, you will just be asked to use this function.

4. Use the derivative function $\boldsymbol{k'(x)}$ to find the *exact* maximum and minimum points of $k(x)$ for **one period**. *(See first page)* Then give general solutions for when these occur.

5. Find the domain and range of $k(x)$. Between the domain and range, which is related to the maximum and minimum points and how?

Part 3 — Questions:

1. Why is $k(0)$ not a maximum or minimum point even though $\boldsymbol{k'(0)} = 0$?
(Hint: Look at the graph.)

2. Suppose a graph with the same shape as $k(x)$ but with a period of 12 months is a model for umbrella sales (where the output is in thousands). Explain one possible reason why the graph looks the way it does.

3. How might the derivative $\boldsymbol{k'(x)}$ help with the manufacture and sale of the umbrellas?