CHAPTER 3 PROBABILITY: EVENTS AND PROBABILITIES

PROBABILITY: A probability is a number between 0 and 1, inclusive, that states the long-run relative frequency, likelihood, or chance that an outcome will happen.

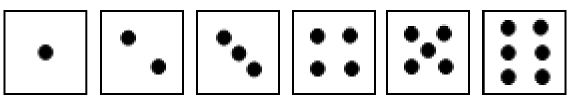
EVENT: An outcome (called a simple event) or a combination of outcomes (called a compound event)

SAMPLE SPACE: Set of all possible simple events

EXAMPLE 1: **Two coins are tossed.**

Assume each coin is a fair coin - it has equal probability of landing on Head (H) or Tail (T). Write the sample space and find the probability that at least one head is obtained.

EXAMPLE 2: **Rolling 1 die: Sample Space: S = { _________________________ }**



Event	2 or 4	even	number ≤ 4	number > 3
Event	D = {2,4}	E = {2, 4, 6}	F = {1, 2, 3, 4}	G={4, 5, 6}
Probability	P(D) =	P(E) =	P(F) =	P(G) =

Compound event: Creating a new event by using **AND, OR, NOT** to relate two or more events

AND: **A and B means BOTH events A and B occur:** *Outcome satisfies both events A and B; includes items in common to both;* ***intersection*** *of A and B*	Event E and F = { } P(E and F) = Event D and G = { } P(D and G) =
OR: **A or B means either event A occurs or event B occurs or both occur** *Outcome satisfies event A or event B or both;* ***union*** *of items from these events.*	Event E or F = { } P(E or F) = Event D or G = { } P(D or G) =
NOT: COMPLEMENT **A′ means event A does NOT occur**	Event D′= { } P(D′) =

CHAPTER 3 PROBABILITY: EVENTS AND PROBABILITIES

COMPLEMENT RULE: For any event A: P(A) + P(A′) = 1   P(A′) = 1 – P(A)

Two events are MUTUALLY EXCLUSIVE if they can NOT both happen: P(A and B) = 0
To check if two events A, B are mutually exclusive, find P(A and B) and see if it is equal to 0.

EXAMPLE 3: Two coins are tossed.

Each coin is a fair coin and has equal probability of landing on Head (H) or Tail (T).

Sample space S = { HH, HT, TH, TT}

Are the events of getting **"two tails"** and getting **"at least one head"** mutually exclusive?

Are the events of getting **"two tails"** and getting **"at most one head"** mutually exclusive?

IF : CONDITIONAL PROBABILITY

Probability that event A occurs IF (given that) we know that outcome B has occurred

P(A|B) = Probability that event A occurs **if** we know that outcome B has occurred

P(A|B) = Probability that event A occurs **"given that"** outcome B has occurred

The vertical line means "if" ; we can also say "given that"

- The event we are interested in comes appears before (to the left of) the "if line"
- The condition is the outcome we know about; it appears after (to the right of) the "if line".

The condition reduces the sample space to be smaller by eliminating outcomes that did not occur.

EXAMPLE 4: Two coins are tossed.

Each coin is a fair coin and has equal probability of landing on Head (H) or Tail (T).
Sample space S = { HH, HT, TH, TT}

Find the probability of getting "**two heads**".

Find the probability of getting "**two heads**" given that **"at least one head"** is obtained.

CHAPTER 3 PROBABILITY: CONDITIONAL PROBABILITY

IF : CONDITIONAL PROBABILITY

Probability that event A occurs IF (given that) we know that outcome B has occurred

P(A|B) = Probability that event A occurs **if** we know that outcome B has occurred

P(A|B) = Probability that event A occurs **"given that"** outcome B has occurred

The vertical line means "if" ; we can also say "given that"

- ♦ The event we are interested in comes appears before (to the left of) the "if line"
- ♦ The condition is the outcome we know about; it appears after (to the right of) the "if line".

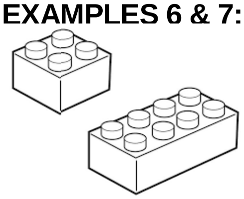
The condition reduces the sample space to be smaller by limiting the sample space to outcomes that we are given information that they occurred and by eliminating outcomes that did not occur.

EXAMPLE 5: Suppose that 10% of students at a college commute by bicycle: P(bicycle) = ________
A bike commute website states that the average speed for a cyclist when commuting is between 10 and 15 miles per hour**.**

Suppose we know that a student lives **30** miles away from the college.
Could that affect the probability that the student commutes by bicycle? How?

Suppose we know that a student lives **3** miles away from the college.
Could that affect the probability that the student commutes by bicycle? How?

EXAMPLES 6 & 7: A box of 25 Lego blocks contains:



2 yellow square blocks	3 yellow rectangular blocks
4 blue square blocks	8 blue rectangular blocks
4 green square blocks	4 green rectangular blocks

Y: yellow
B: blue
G: green
S: square
R: rectangle

EXAMPLE 6: Suppose that a child randomly takes one Lego block from the box.

a. Find the probability that the block is blue: ______________________________

b. Find the probability that the block is blue given that (if) it is square: ______________________________

c. Find the probability that the block is square given that (if) it is blue: ______________________________

OBSERVATION #1: In general, for two events A, B: P(A|B) ≠ P(B|
Order Matters! We need to be careful which is the "event" and which is the "condition"
This is different from "and" or "or" events where the order in which it is written does not matter.

EXAMPLE 7: If one block is randomly picked from the box of Lego blocks:

a. Find the probability that the block is yellow : ______________________________

b. Find the probability that the block is yellow given that (if) it is square: ______________________________

OBSERVATION #2: Sometimes knowing the condition occurred changes the probability of the event,
BUT sometimes knowing the condition occurred does not affect the probability of the event.

INDEPENDENT EVENTS:

Two events are independent if and only if the probability of one event (A) occurring is not affected by whether the other event (B) occurs or not.

Events A and B are independent if P(A) = P(A|B).

- knowing that B occurs does not change the probability of A occurring
- P(event) = P (event | condition)

EXAMPLE 8: Source: http://www.indexmundi.com/blog/index.php/2013/06/25/male-and-female-literacy-rates-by-country/

In Argentina, the literacy rate is 97% for men and 97% for women.
The overall literacy rate is 97%.
Is the literacy rate in Argentina independent of gender? *Justify your answer using appropriate probabilities.*
Events: F = female M = male L = literate

EXAMPLE 9: Source: http://www.censusindia.gov.in/2011-prov-results/indiaatglance.html

In India, literacy rates are 82.1% for men and 65.5% for women
The overall literacy rate is estimated as approximately 74%.
Is the literacy rate in India independent of gender? *Justify your answer using appropriate probabilities.*
Events: F = female M = male L = literate

Note: The literacy rates in India have improved, overall, and particularly for females, the gap is closing:
2011 literacy rates: Overall 74% Male: 82.1% Female: 65.5%
2001 literacy rates: Overall 64.8% Male: 75.3% Female: 53.7%

TO CHECK IF TWO EVENTS ARE INDEPENDENT in a word problem

- Identify the probabilities you are given by reading the problem carefully
- See which is a conditional probability: P(event|condition)
- Compare it to probability of same event without the condition: P(event)
- If P(event) = P(event|condition) they are independent

Note: there are other ways to check for independence, discussed in the textbook.
In Mrs. Bloom's class this way almost always is easiest.

CHAPTER 3 PROBABILITY: INDEPENDENT EVENTS

EXAMPLE 10: In a sample of 100 students at a community college, 60 were full time and 40 were part-time. 33 of the full time students intend to transfer. 10 of the part time students intend to transfer.

Events: F = fulltime T = transfer

Find the probability that a student intends to transfer.

Find the probability that a student intends to transfer given that (if) the student is full time.

Are the two events "intend to transfer" and "full-time" independent events?

Clearly state your conclusion and use probabilities to justify your conclusion.

EXAMPLE 11: In a math class of 50 students,

80% of all students passed a quiz.
60% of students use the print textbook
40% of students use the ebook.
Of the 20 students who use the ebook, 16 of them passed the quiz

Events: Q = student passed the quiz E = student uses ebook T = student uses print textbook

Are events Q and E independent?

Justify your answer using appropriate probabilities

EXAMPLE 12 PRACTICE:

Big Shoe Wearhouse finds that

40% of their shoe sales are online on the website.
60% of their shoe sales are in the store
15% of all shoes purchased are returned
Of the shoe purchases made online, 25% are returned.

Events: S = purchased in store W = purchased on website R = item is returned

Are events R and W independent?

Justify your answer using appropriate probabilities.

CHAPTER 3: PROBABILITY in CONTINGENCY TABLES

A contingency table displays data for two variables. This table shows the number of individuals or items in each category. We can use the data in the table to find probabilities.

All probabilities <u>EXCEPT conditional probabilities</u> have the grand total in the denominator

<u>Conditional Probabilities</u>: The condition limits you to a particular row or column in the table.
Condition says "IF" we look only at a particular row or column, find the probability
The **denominator will be the total for the row or column** in the table that corresponds to the condition

EXAMPLE 13: A large car dealership examined a sample of vehicles sold or leased in the past year. Data is classified by type (**car, SUV, van, truck**) and by whether they were a sale of a **new** or **used** vehicle or whether the vehicle was **leased**.

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale (N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Suppose a vehicle in the sample is randomly selected to review its sales or lease papers.

a. Find the probability that the vehicle was leased.

b. Find the probability that a vehicle is a truck.

c. Find the probability that a vehicle is NOT a truck.

d. Find the probability that the vehicle was a car AND was leased.

e. Find the probability that a vehicle was used GIVEN THAT it was a van.

f. Find the probability that the vehicle was a van GIVEN THAT it was used.

Addition Rule for OR Events: P(A or B) = P(A) + P(B) – P(A and B)

g. Find the probability that the vehicle was used OR a van.

h. Find the probability that the vehicle was leased OR a truck.

Mutually Exclusive Events: P(A and B) = 0

i. Are events T, L mutually exclusive?

j. Are events S, U mutually exclusive?

CHAPTER 3: PROBABILITY and INDEPENDENCE in CONTINGENCY TABLES

An easy way to check if two events are independent in a contingency table is

Let the column be the "condition"

Let the row be the "event"

Compare : P(row event | condition in column) to P(row event using total column)

If and only if these probabilities are equal, then the events are independent

EXAMPLE 14: Are the events N and V independent?

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale(N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

P(Event | Condition) = P(_____ | ______) = P(Event) = P(_____) =

Conclusion:___________________________________ Reason ___________________

EXAMPLE 15: Are the events S and U independent?

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale(N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

EXAMPLE 16 PRACTICE:

Suppose that a sample residents of a town with a large university gave the data below:

	(S) College Student	(N) Not College Student	TOTAL
(A) Amazon Prime Member	40	20	60
(B) Not Amazon Prime Member	60	130	190
TOTAL	100	150	250

Are events of “student” and “Amazon Prime member” independent?

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

CHAPTER 3: PROBABILITY TREES

TREE DIAGRAMS are a useful tool in organizing and solving probability problems
Each complete path through the tree represents a separate mutually exclusive outcome in the sample space.

1. Draw a tree representing the possible mutually exclusive outcomes
2. Assign conditional probabilities along the branches of the tree
3. Multiply probabilities along each complete path through the tree to find probabilities of each "AND" outcome in the sample space.
4. Add probabilities for the appropriate paths of a tree to find the probability of a compound OR event.

EXAMPLE 17: *From Chapter 3 Section 3.7 Tree diagrams in Illowsky, B., & Dean, S. Collaborative Statistics. Connexions, Dec. 5, 2008. http://cnx.org/content/col10522/1.29*

An urn contains 11 marbles, 3 Yellow and 8 Blue. We are selecting 2 marbles randomly from the urn.
Draw a tree diagram. Show the events and probabilities for each branch and each complete path of the tree.

Select 2 marbles WITH REPLACEMENT:
Find the probability of selecting one marble of each color

Select 2 marbles WITHOUT REPLACEMENT
Find the probability of selecting one marble of each color

CHAPTER 3: PROBABILITY TREES

EXAMPLE 18: A certain virus infects 10% of people
A test used to detect the virus can give a positive result or a negative result.
The test results are positive 80% of the time IF the person has the virus
For people who do not have the virus, the test results are positive 5% of the time ("false positive")

V = event that a person has the virus
Pos = event that the test is positive **Neg** = event that the test is negative

Do ALL of the following:
(a) Fill in all probabilities along the branches of the tree.
(b) Find and write the event and probability corresponding to each complete path through the tree.
(c) Find the probability a person's test result is positive. *Show your work.*
Round all probabilities to 3 decimal places (thousandths)

EXAMPLE 19 PRACTICE: Suppose we toss a biased coin twice. A biased coin is a coin that is not fair.
If you toss this coin, it lands on Heads 40% of the time and lands on Tails 60% of the time.
Use a tree to find the probabilities of each outcome in the sample space.

CHAPTER 3: PROBABILITY RULES:

♦ **Complement Rule:** **$P(A') = 1 - P(A)$**

♦ **Addition Rule for <u>OR</u> Events:** **$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$**

IF AND ONLY IF events are MUTUALLY EXCLUSIVE: $P(A \text{ or } B) = P(A) + P(B)$

♦ **Multiplication Rule for <u>AND</u> Events:** **$P(A \text{ and } B) = P(A \mid B)\, P(B)$**

IF AND ONLY IF events are INDEPENDENT: $P(A \text{ and } B) = P(A)\, P(B)$

♦ **Conditional Probability Rule:** $P(A|B) = \dfrac{P(A \text{ AND } B)}{P(B)} = \dfrac{\textit{probability of ``and'' event}}{\textit{probability of condition}}$

("given that", "if")

EXAMPLE 20: Addition Rule for OR Events; Conditional Probability Rule

In a certain town: 70% of households have Cable TV (*event C*)
55% of households have Netflix (*event N*)
These figures include the fact that 42% of households subscribe to both.

a . Find the probability that a person subscribes to Cable TV **or** Netflix

b. Find the probability that a household subscribes to Netflix **given that** the household has Cable TV

c. Find the probability that a household has Cable TV **if** the household subscribes to Netflix

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EXAMPLE 21: PRACTICE: Addition Rule for OR Events; Conditional Probability Rule

Big Shoe Wearhouse is concerned about customer satisfaction with online purchases.
40% of all shoe sales are online on their website.
60% of all shoe sales are in their stores.
Overall, 15% of all shoe purchases are returned.
10% of all shoe purchases were made on the website **and** were returned.
Events: S = purchased in store   W = purchased on website   R = item is returned

a. Find the probability that a shoe purchase was made on the website **or** it was returned

b. Find the probability that a shoe purchase was made on the website **given that** it is returned.

c. Find the probability that a shoe purchase is returned **if** it was purchased on the website

CHAPTER 3: PROBABILITY RULES:

♦**Complement Rule:** **P(A′) = 1 – P(A)**

♦**Addition Rule for <u>OR</u> Events:** **P(A or B) = P(A) + P(B) – P(A and B)**

IF AND ONLY IF events are MUTUALLY EXCLUSIVE: P(A or B) = P(A) + P(B)

♦**Multiplication Rule for <u>AND</u> Events:** **P(A and B) = P(A | B) P(B)**

IF AND ONLY IF events are INDEPENDENT: P(A and B) = P(A) P(B)

♦**Conditional Probability Rule:** $P(A|B) = \dfrac{P(A \text{ AND } B)}{P(B)} = \dfrac{\textit{probability of “and” event}}{\textit{probability of condition}}$

(“given that”, “if”)

EXAMPLE 22: Multiplication Rule for AND Events

T = student intends to transfer
S = student takes statistics

At a college: 45% of all students take Statistics.
70% of all students intend to Transfer.
Of the students who intend to Transfer, 60% of them take Statistics.

Find the probability that a student intends to Transfer AND takes Statistics.

EXAMPLE 23: Multiplication Rule for Independent AND Events

In a math class, 75% of students pass the quiz (*event Q*).
60% of students use a print textbook (*event T*) and 40% use the e-book (*event E*).
Based on data she collected, the instructor has determined that whether a student passes the quiz **is independent** of whether the student uses the book as an ebook.

Find the probability that a student uses the e-book AND passes the quiz.

EXAMPLE 24: PRACTICE: Multiplication Rule for AND Events

At a fast food restaurant: 75% of customers order burgers (*event B*)
70% of customers order fries (*event F*)
Of the customers who order burgers, 80% also order fries.
Find the probability that a customer orders both a burger and fries.

EXAMPLE 25: PRACTICE: Multiplication Rule for AND Events

(Hint: Read carefully to understand information given in the "story")

For job listings on a job posting website : 30% require professional certification (*event C*)
65% require a college degree (*event D*)
50% require 5+ years of related job experience (*event E*)
14% of job listings requiring a college degree also require professional certification.
Find the probability that a job requires both professional certification and a college degree.