Practice Problems: Setting Up Linear Programs with Mixed Constraints

For each problem, write the linear program.
State whether you are maximizing or minimizing the objective function.
State the objective function.
Write all constraints.
DO NOT SOLVE.

If you insist on solving, question #1 can be solved graphically using variables x and y instead of x1 and x2.

You can not solve questions #1 or #2 using the simplex method as we are learning it in our class.
We are not learning how to extend the simplex method to problems with mixed constraints.
If you want to solve these problems, use technology - a simplex method solver tool. There are many such solvers available; one such link is posted on the Chapter 3 & 4 Linear Programming Resource Webpage for this class. There are other such tools also, including apps for phones and tablets.

1. Flickx Company streams original content movies and TV shows.
Producing each TV show episode requires $100,000 in capital and 400 hours of staff time.
Producing each movie requires $1,000,000 in capital and 2,000 hours of staff time.
The production budget has at most $6,000,000 in capital and can support at most 16,000 hours of staff time.
The marketing staff requires that at least 12 TV episodes are needed.
On average each TV show episode earns $400,000 and each movie earns $3,500,000.

\[ x_1 = \text{number of TV show episodes} \]
\[ x_2 = \text{number of movies} \]

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Cite as Finite Math Resource for Math 11 at De Anza College, 2019, deanza.edu/faculty/bloomroberta

2. A company is creating a meal replacement bar. They plan to use peanut butter, oats, and dried cranberries as the primary ingredients. The nutritional content per gram of each is listed below, along with the cost, in cents per gram, of each ingredient.

\[ x_1 = \text{number of 1g servings of peanut butter} \]
\[ x_2 = \text{number of 1g servings of oats} \]
\[ x_3 = \text{number of 1g servings of cranberries} \]

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Peanut Butter per gram</th>
<th>Oats per gram</th>
<th>Cranberries per gram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protein (grams)</td>
<td>0.25</td>
<td>0.17</td>
<td>0</td>
</tr>
<tr>
<td>Fat (grams)</td>
<td>0.5</td>
<td>0.07</td>
<td>0.01</td>
</tr>
<tr>
<td>Cost (cents)</td>
<td>0.6</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Find the amount of each ingredient they should use to minimize the cost of producing a bar containing at least 15g of each ingredient, at least 10g of protein and at most 14g of fat.

Problem 2 on this page and its solution on the next page is adapted from Business Precalculus © David Lippman 2016.
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Solutions:

1. Maximize \( z = 400000 x_1 + 3500000 x_2 \)
   Subject to
   \( 100000 x_1 + 100000 x_2 \leq 6000000 \)
   \( 400 x_1 + 2000 x_2 \leq 16000 \)
   \( x_1 \geq 12 \)
   \( x_2 \geq 0 \)

   Optimal Solution: \( z = 22,000,000; x_1 = 20 \) TV episodes, \( x_2 = 4 \) movies
   They should produce 20 TV episodes and 4 movies to maximize profit at $22,000,000.
   Using online Simplex Solver Tool: https://www.zweigmedia.com/RealWorld/simplex.html

2. Minimize \( z = 0.6 x_1 + 0.1 x_2 + 0.2 x_3 \)
   Subject to:
   \( 0.25 x_1 + 0.17 x_2 \geq 10 \)
   \( 0.5 x_1 + 0.07 x_2 + 0.01 x_3 \leq 14 \)
   \( x_1 \geq 15 \)
   \( x_2 \geq 15 \)
   \( x_3 \geq 15 \)

   Optimal Solution: \( z = 15.6765; x_1 = 15, x_2 = 36.7647, x_3 = 15 \)
   Interpreting that result, the minimum cost of to produce the bar will be about 15.7 cents, by using
   15 grams of peanut butter, 36.8 grams of oats, and 15 grams of dried cranberries.

**NOTE:** IN GENERAL EVERY LINEAR PROGRAM WE ENCOUNTER IN OUR MATH 11 CLASS NEEDS NON-NEGATIVITY CONSTRAINTS FOR EACH VARIABLE.
The only time you do not need to write the non-negativity constraint for some variable (such as \( x_1 \geq 0 \)) is when there is another constraint on the same variable that requires that variable to be at least (greater than or equal to) some positive non-zero value (such as \( x_1 \geq 15 \)).
If you are not sure if it is needed or not, it never hurts to write the non-negativity constraint just to be safe that you have not omitted something that is needed.